

Method of variation of parameters to solve
L.D.E. of 2nd order with variable coefficients.

Given $a_0(x)y'' + a_1(x)y' + a_2(x)y = f(x)$ — (1)

c.f. of (1) $y_c(x) = C_1 y_1(x) + C_2 y_2(x)$

Then p.i. of (1) $y_p = \underline{v_1(x) y_1(x)} + \underline{v_2(x) y_2(x)}$

$$v_1(x) = - \int \frac{f(x) \cdot y_2}{a_0(x) W(y_1, y_2)} dx \quad v_2(x) = \int \frac{f(x) \cdot y_1}{a_0(x) W(y_1, y_2)} dx$$

Q.1. $y'' + y = \tan x$

$$\frac{d^2 y}{dx^2} + y = \tan x$$

$$(D^2 + 1)y = 0$$

A.E. $m^2 + 1 = 0$

$$m = \pm i$$

$$y_c = e^{0x} (C_1 \cos x + C_2 \sin x)$$

$$= C_1 \cos x + C_2 \sin x$$

$$= C_1 y_1(x) + C_2 y_2(x)$$

$$y_1(x) = \cos x, \quad y_2(x) = \sin x$$

$$y_p = v_1(x) y_1(x) + v_2(x) y_2(x)$$

$$= \underline{v_1(x) \cos x} + \underline{v_2(x) \sin x}$$

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$$

$$v_1(x) = - \int \frac{f(x) y_2(x)}{a_0(x) W(y_1, y_2)} dx = - \int \frac{\tan x \cdot \sin x}{1 \cdot 1} dx$$

$$= - \int \frac{\sin^2 x}{\cos x} dx = + \int \frac{-1 + \cos^2 x}{\cos x} dx$$

$$= - \int \sec x dx + \int \cos x dx$$

$$= -\log |\sec x + \tan x| + \sin x$$

$$v_2(x) = \int \frac{f(x) y_1(x)}{a_0(x) w(y_1, y_2)} dx = \int \frac{\tan x \cdot \cos x}{1 \cdot 1} dx$$

$$= \int \sin x dx = -\cos x$$

$$y_p = (\sin x - \log |\sec x + \tan x|) \cos x - \sin x \cos x$$

$$= -\cos x \log |\sec x + \tan x|$$

$$\text{G.S. } y = C_1 \cos x + C_2 \sin x - \cos x \cdot \log |\sec x + \tan x| \quad \underline{\text{Ans.}}$$

$$\text{Q.2.} \quad (D^2 + 4D + 4)y = e^{-2x} / x^2$$

$$\text{A.E.} \quad m^2 + 4m + 4 = 0$$

$$(m+2)^2 = 0$$

$$m = -2, -2$$

$$y_c = (C_1 + C_2 x) e^{-2x}$$

$$= C_1 e^{-2x} + C_2 x e^{-2x}$$

$$= C_1 y_1 + C_2 y_2$$

$$y_1 = e^{-2x} \quad y_2 = x e^{-2x}$$

$$y_p = v_1 y_1 + v_2 y_2$$

$$= v_1 e^{-2x} + v_2 \cdot x e^{-2x}$$

$$w(y_1, y_2) = \begin{vmatrix} e^{-2x} & x e^{-2x} \\ -2e^{-2x} & (1-2x)e^{-2x} \end{vmatrix} = e^{-4x}$$

$$v_1 = - \int \frac{f(x) y_2}{a_0 \cdot w} dx = - \int \frac{e^{-2x} \cdot x e^{-2x}}{x^2 \cdot e^{-4x}} dx$$

$$= - \int \frac{1}{x} dx = -\log x$$

$$v_2 = \int \frac{f(x) y_1}{a_0 \cdot w} dx = \int \frac{e^{-2x} \cdot e^{-2x}}{x^2 \cdot e^{-4x}} dx = \int \frac{1}{x^2} dx$$

$$v_2 = \int \frac{f(x) \cdot u'}{a_0 \cdot w} dx = \int \frac{e^{-2x} \cdot (-1/x)}{x^2 \cdot e^{-4x}} dx = - \int \frac{1}{x^3} dx$$

$$y_p = -\log x \cdot e^{-2x} + \left(-\frac{1}{x}\right) x \cdot e^{-2x}$$

$$= -e^{-2x} (1 + \log x)$$

G.S. $y = (C_1 + C_2 x) e^{-2x} - e^{-2x} (1 + \log x)$

Ans.

Q-9. $(D^2 + a^2)y = \sec ax$

Ans. $C_1 \cos ax + C_2 \sin ax + \frac{1}{a^2} \cos ax \log \cos ax + \frac{x}{a} \sin ax$